Name:_____ Class:_____ Date:_____

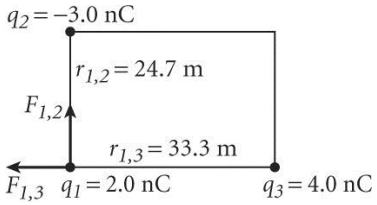
Electric Forces and Fields

Problem B

THE SUPERPOSITION PRINCIPLE

PROBLEM

The cinema screen installed at the Science Park, in Taejon, Korea, is 24.7 m high and 33.3 m wide. Consider the arrangement of charges shown below. If q_1 = 2.00 nC, q_2 = –3.00 nC, and q_3 = 4.00 nC, find the magnitude and direction of the resultant electric force on q_1.



REASONING

According to the superposition principle, the resultant force on the charge q_1 is the vector sum of the forces exerted by q_2 and q_3 on q_1. First find the force exerted on q_1 by each charge, then use the Pythagorean theorem to find the magnitude of the resultant force on q_1. Take the ratio of the resultant *y* component to the resultant *x* component, and then take the arc tangent of that quantity to find the direction of the resultant force on q_1.

SOLUTION

Given: $q_1 = 2.00 \text{ nC} = 2.00 \times 10^{-9} \text{ C}$

$q_2 = -3.00 \text{ nC} = -3.00 \times 10^{-9} \text{ C}$

$q_3 = 4.00 \text{ nC} = 4.00 \times 10^{-9} \text{ C}$

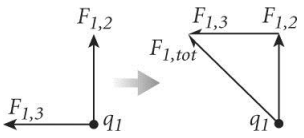
$r_{1,2} = 24.7 \text{ m}$

$r_{1,3} = 33.3 \text{ m}$

$k_C = 8.99 \times 10^9 \text{ N} \bullet \text{m}^2/\text{C}^2$

Unknown: $F_{1,tot} = ?$

Diagram:



1. **Calculate the magnitude of the forces with Coulomb's law:**

$$F_{2,1} = k_C \frac{q_2 q_1}{(r_{2,1})^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \bullet \text{m}^2}{\text{C}^2}\right)\left[\frac{(3.00 \times 10^{-9} \text{ C})(2.00 \times 10^{-9} \text{ C})}{(24.7 \text{ m})^2}\right]$$

$$F_{2,1} = 8.84 \times 10^{-11} \text{ N}$$

$$F_{3,1} = k_C \frac{q_3 q_1}{(r_{3,1})^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \bullet \text{m}^2}{\text{C}^2}\right)\left[\frac{(4.00 \times 10^{-9} \text{ C})(2.00 \times 10^{-9} \text{ C})}{(33.3 \text{ m})^2}\right]$$

$$F_{3,1} = 6.49 \times 10^{-11} \text{ N}$$

2. **Determine the direction of the forces by analyzing the signs of the charges:**

   The force $\mathbf{F_{2,1}}$ is attractive because q_1 and q_2 have opposite signs. $\mathbf{F_{2,1}}$ is directed along the positive *y*-axis, so its sign is positive.

   The force $\mathbf{F_{3,1}}$ is repulsive because q_1 and q_3 have the same sign. $\mathbf{F_{3,1}}$ is directed toward the negative *x*-axis, so its sign is negative.

3. **Find the *x* and *y* components of each force:**

   For $\mathbf{F_{2,1}}$: $F_x = F_{3,1} = -6.49 \times 10^{-11}$ N; $F_y = 0$

   For $\mathbf{F_{3,1}}$: $F_y = F_{2,1} = 8.84 \times 10^{-11}$ N; $F_x = 0$

4. **Calculate the magnitude of the total force acting in both directions:**

$$F_{x,tot} = F_x = -6.49 \times 10^{-11} \text{ N}$$
$$F_{y,tot} = F_y = 8.84 \times 10^{-11} \text{ N}$$

5. **Use the Pythagorean theorem to find the magnitude of the resultant force:**

$$F_{1,tot} = \sqrt{(F_{x,tot})^2 + (F_{y,tot})^2} = \sqrt{(-6.49 \times 10^{-11}\ \text{N})^2 + (8.84 \times 10^{-11}\ \text{N})^2}$$

$$F_{1,tot} = 1.10 \times 10^{-10}\ \text{N}$$

6. **Use a suitable trigonometric function to find the direction of the resultant force:**

   In this case, you can use the inverse tangent function.

$$\tan\theta = \frac{F_{y,tot}}{F_{x,tot}} = \frac{(8.84 \times 10^{-11}\ \text{N})}{(-6.49 \times 10^{-11}\ \text{N})} = -1.36$$

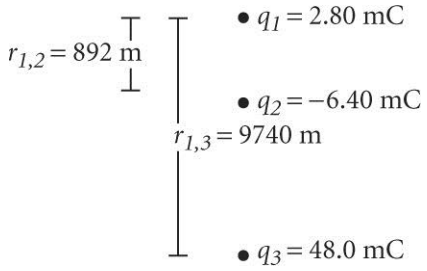
$$\theta = \tan^{-1}(-1.36) = -53.7°$$

7. **Evaluate your answer:**

   The resultant force makes an angle of 53.7° to the left and above the *x*-axis.

ADDITIONAL PRACTICE

1. **In 1919 in Germany, a train of eight kites was flown 9740 m above the ground. This distance is 892 m higher than Mount Everest. Consider the arrangement of charges located at the various heights shown below. If q_1 = 2.80 mC, q_2 = –6.40 mC, and q_3 = 48.0 mC, find the magnitude and direction of the resultant electric force acting on q_1.**



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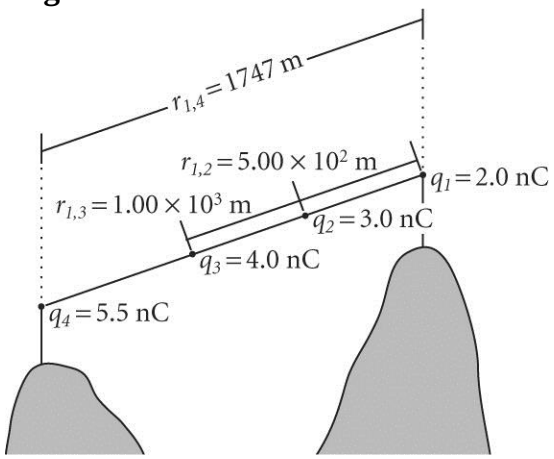
2.In 1994, a group of British and Canadian athletes performed a rope slide off the top of Mount Gibraltar, in Canada. The speed of the sliders at times exceeded 160 km/h. The total length of the slide was 1747 m. Suppose several sliders are located on the rope as shown. Due to friction, they acquire the electric charges shown. Find the magnitude and direction of the resultant electric force acting on the athlete at the far right of the diagram.



3. In 1913, a special postage stamp was issued in China. It was 248 mm long and 70.0 mm wide. Suppose equal charges of 1.0 nC are placed in the corners of this stamp. Find the magnitude and direction of the resultant electric force acting on the upper right corner (assume the widest part of the stamp is aligned with the *x*-axis).
4. In 1993, a chocolate chip cookie was baked in Arcadia, California. It contained about three million chips and was 10.7 m long and 8.7 m wide. Suppose four charges are placed in the corners of that cookie as follows: $q_1 = -12.0$ nC at the lower left corner, $q_2 = 5.6$ nC at the upper left corner, $q_3 = 2.8$ nC at the upper right corner, and $q_4 = 8.4$ nC at the lower right corner. Find the magnitude and direction of the resultant electric force acting on q_1.
5. In 1988, a giant firework was exploded at the Lake Toya festival, in Japan. The shell had a mass of about 700 kg and produced a fireball 1.2 km in diameter. Consider a circle with this diameter. Suppose four charges are placed on the circle's perimeter so that the lines between them form a square with sides parallel to the *x*- or *y*-axes. The charges have the following strengths and locations: $q_1 = 16.0$ mC at the upper left "corner," $q_2 = 2.4$ mC at the upper right corner, $q_3 = -3.2$ mC at the lower right corner, and $q_4 = -4.0$ mC at the lower left corner. Find the magnitude and direction of the resultant electric force acting on q_1. (Hint: Find the distances between the charges first.)
6. American athlete Jesse Castenada walked 228.930 km in 24 h in 1976, setting a new record. Consider an equilateral triangle with a perimeter equal to the distance Castenada walked. Suppose the charges are placed at the following vertices of the triangle: $q_1 = 8.8$ nC at the bottom left vertex, $q_2 = -2.4$ nC at the bottom right vertex, and $q_3 = 4.0$ nC at the top vertex. Find the magnitude and direction of the resultant electric force acting on q_1.