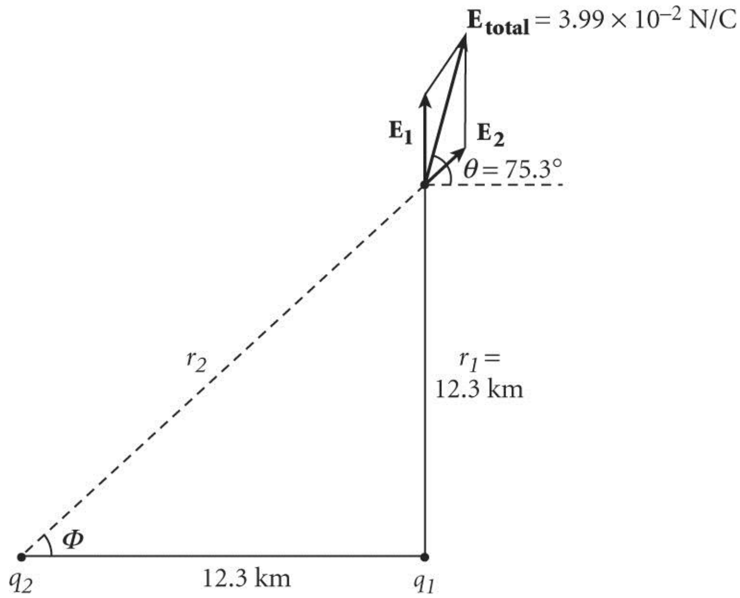
Electric Forces and Fields

Problem D

ELECTRIC FIELD STRENGTH

PROBLEM

The Seto-Ohashi bridge, linking the two Japanese islands of Honshu and Shikoku, is the longest “rail and road” bridge, with an overall length of 12.3 km. Suppose two equal charges are placed at the opposite ends of the bridge. If the resultant electric field strength due to these charges at the point exactly 12.3 km above one of the bridge’s ends is 3.99×10^{-2} N/C and is directed at 75.3° above the positive *x*-axis, what is the magnitude of each charge?



REASONING

According to the superposition principle, the resultant electric field strength at the point above the bridge is the vector sum of the electric field strengths produced by q_1 and q_2. First, find the components of the electric field strengths produced by each charge, then combine components in the *x* and *y* directions to find the electric field strength components of the resultant vector. Equate this to the components in the *x* and *y* directions of the electric field vector. Finally, rearrange the equation to solve for the charge.

SOLUTION

Given: $E_{tot} = 3.99 \times 10^{-4}$ N/C

$\theta = 75.3°$

$r_1 = 12.3 \text{ km} = 1.23 \times 10^4 \text{ m}$

$k_C = 8.99 \times 10^9 \text{ N•m}^2/\text{C}^2$

Unknown: $q_1 = ?$ $q_2 = ?$

The distance r_2 must be calculated from the information in the diagram. Because r_2 forms the hypotenuse of a right triangle whose sides equal r_1, it follows that

$$r_2 = \sqrt{(r_1)^2 + (r_1)^2} = \sqrt{2(r_1)^2} = 1.74 \times 10^4 \text{ m}$$

The angle that r_2 makes with the coordinate system equals the inverse tangent of the ratio of the vertical to the horizontal components. Because these components are equal,

$$\tan \phi = 1.00, \text{ or } \phi = 45.0°$$

1. **Find the *x* and *y* components of each electric field strength vector:**

   At this point, the direction of each component must be taken into account.

   For $\mathbf{E_1}$: $E_{x,1} = 0$

   $$E_{y,1} = E_1 = \frac{k_C q_1}{(r_1)^2}$$

   For $\mathbf{E_2}$: $E_{x,2} = E_2 \cos(45.0°) = \dfrac{k_C q_2}{\sqrt{2}(r_2)^2}$

   $$E_{y,2} = E_2 \sin(45.0°) = \frac{k_C q_2}{\sqrt{2}(r_2)^2}$$

2. **Calculate the magnitude of the total electric field strength in both the *x* and *y* directions:**

   $$E_{x,tot} = E_{x,1} + E_{x,2} = \frac{k_C q_2}{\sqrt{2}(r_2)^2} = E_{tot} \cos(75.3°)$$

   $$E_{y,tot} = E_{y,1} + E_{y,2} = \frac{k_C q_1}{(r_1)^2} + \frac{k_C q_2}{\sqrt{2}(r_2)^2} = E_{tot} \sin(75.3°)$$

5. **Rearrange the equation(s) to isolate the unknown(s):**

$$q_2 = \frac{E_{tot} \cos(75.3°)\sqrt{2}(r_2)^2}{k_C} = \frac{(3.99 \times 10^{-2} \text{ N/C})\cos(75.3°)\sqrt{2}(1.74 \times 10^4 \text{ m})^2}{8.99 \times 10^9 \text{ N} \bullet \text{m}^2/\text{C}^2}$$

$$q_2 = q_1 = 4.82 \times 10^{-4} \text{ C}$$

ADDITIONAL PRACTICE

Lots of Equation Manipulation

1. **The world's largest tires have a mass of almost 6000 kg and a diameter of 3.72 m each. Consider an equilateral triangle with sides that are 3.72 m long each. If equal positive charges are placed at the points on either end of the triangle's base, what is the direction of the resultant electric field strength vector at the top vertex? If the magnitude of the electric field strength at the top vertex equals 0.145 N/C, what are the two quantities of charge at the base of the triangle?**
2. **The largest fountain is found at Fountain Hills, Arizona. Under ideal conditions, the 8000 kg column of water can reach as high as 190 m. Suppose a 12 nC charge is placed on the ground and another charge of unknown quantity is located 190 m above the first charge. At a point on the ground 120 m from the first charge, the horizontal component of the resultant electric field strength is found to be $E_x = 1.60 \times 10^{-2}$ N/C. Using this information, calculate the unknown quantity of charge.**
3. Pontiac Silverdome Stadium, in Detroit, Michigan, is the largest air-supported building in the world. Suppose a charge of 18.0 μC is placed at one end of the stadium and a charge of –12.0 μC is placed at the other end of the stadium. If the electric field halfway between the charges is 22.3 N/C, directed toward the –12.0 μC charge, what is the length of the stadium?

Lots of Equation Manipulation

4. **In 1897, a Ferris wheel with a diameter of 86.5 m was built in London. The wheel held 10 first-class and 30 second-class cabins, and each cabin was capable of carrying 30 people. Consider two cabins positioned exactly opposite each other. Suppose one cabin has an unbalanced charge of 4.8 nC and the other cabin has a charge of 16 nC. At what distance from the 4.8 nC charge along the diameter of the wheel would the strength of the resultant electric field be zero?**
5. Suppose three charges of 3.6 μC each are placed at three corners of the Imperial Palace in Beijing, China, which has a length of 960 m and a width of 750 m. What is the strength of the electric field at the fourth corner?
6. The world's largest windows, which are in the Palace of Industry and Technology in Paris, France, have a maximum width of 218 m and a maximum height of 50.0 m. Consider a rectangle with these dimensions. If charges are placed at its corners, as shown in the figure below, what is the electric field strength at the center of the rectangle? The value of q is 6.4 nC.