Name:______________________________ Class:__________________ Date:__________________

Two-Dimensional Motion and Vectors

Problem C

ADDING VECTORS ALGEBRAICALLY

PROBLEM

The record for the longest nonstop closed-circuit flight by a model airplane was set in Italy in 1986. The plane flew a total distance of 1239 km. Assume that at some point the plane traveled 1.25×10^3 m to the east, then 1.25×10^3 m to the north, and finally 1.00×10^3 m to the southeast. Calculate the total displacement for this portion of the flight.

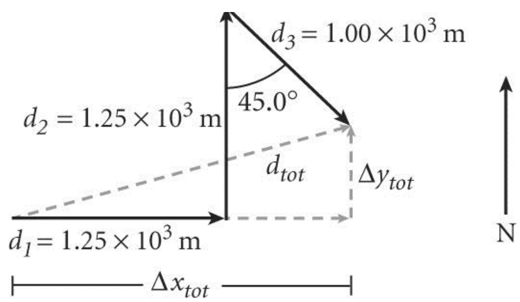
SOLUTION

1. **DEFINE**

Given: $d_1 = 1.25 \times 10^3$ m $\quad d_2 = 1.25 \times 10^3$ m $\quad d_3 = 1.00 \times 10^3$ m

Unknown: $\Delta x_{tot} = ?$ $\Delta y_{tot} = ?$ $\quad d = ?$ $\quad \theta = ?$

Diagram:



2. **PLAN** **Choose the equation(s) or situation:** Orient the displacements with respect to the x-axis of the coordinate system.

$\theta_1 = 0.00°$ $\qquad \theta_2 = 90.0°$ $\qquad \theta_3 = -45.0°$

Use this information to calculate the components of the total displacement along the x-axis and the y-axis.

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 + \Delta x_3$$
$$= d_1(\cos\theta_1) + d_2(\cos\theta_2) + d_3(\cos\theta_3)$$
$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 + \Delta y_3$$
$$= d_1(\sin\theta_1) + d_2(\sin\theta_2) + d_3(\sin\theta_3)$$

Use the components of the total displacement, the Pythagorean theorem, and the tangent function to calculate the total displacement.

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} \qquad \theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right)$$

3. **CALCULATE** **Substitute the values into the equation(s) and solve:**

$$\Delta x_{tot} = (1.25 \times 10^3 \text{ m})(\cos 0°) + (1.25 \times 10^3 \text{ m})(\cos 90.0°)$$
$$+ (1.00 \times 10^3 \text{ m})[\cos(-45.0°)]$$
$$= 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m}$$
$$= 1.96 \times 10^3 \text{ m}$$
$$\Delta y_{tot} = (1.25 \times 10^3 \text{ m})(\sin 0°) + (1.25 \times 10^3 \text{ m})(\sin 90.0°)$$
$$+ (1.00 \times 10^3 \text{ m})[\sin(-45.0°)]$$
$$= 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m}$$
$$= 0.543 \times 10^3 \text{ m}$$

$$d = \sqrt{(1.96 \times 10^3\ \text{m})^2 + (0.543 \times 10^3\ \text{m})^2}$$

$$d = \sqrt{3.84 \times 10^6\ \text{m}^2 + 2.95 \times 10^5\ \text{m}^2} = \sqrt{4.14 \times 10^6\ \text{m}^2}$$

$$d = 2.03 \times 10^3\ \text{m}$$

$$\theta = tan^{-1}\left(\frac{0.543 \times 10^3\ \text{m}}{1.96 \times 10^3\ \text{m}}\right)$$

$$\theta = 15.5° \text{ north of east}$$

4. **EVALUATE**

The magnitude of the total displacement is slightly larger than that of the total displacement in the eastern direction alone.

ADDITIONAL PRACTICE

1. **For six weeks in 1992, Akira Matsushima, from Japan, rode a unicycle more than 3000 mi across the United States. Suppose Matsushima is riding through a city. If he travels 250.0 m east on one street, then turns counterclockwise through a 120.0° angle and proceeds 125.0 m northwest along a diagonal street, what is his resultant displacement?**
2. **In 1976, the Lockheed SR-71A *Blackbird* set the record speed for any airplane: 3.53×10^3 km/h. Suppose you observe this plane ascending at this speed. For 20.0 s, it flies at an angle of 15.0° above the horizontal, then for another 10.0 s its angle of ascent is increased to 35.0°. Calculate the plane's total gain in altitude, its total horizontal displacement, and its resultant displacement.**
3. Magnor Mydland of Norway constructed a motorcycle with a wheelbase of about 12 cm. The tiny vehicle could be ridden at a maximum speed of 11.6 km/h. Suppose this tiny motorcycle travels in the directions d_1 and d_2, where d_1 is 30˚ with the horizontal (upward and right) and d_2 is 45˚ with the vertical (down and to the right). Calculate d_1 and d_2, and determine how long it takes the motorcycle to reach a net displacement of 2.0×10^2 to the right.
4. The fastest propeller-driven aircraft is the Russian TU-95/142, which can reach a maximum speed of 925 km/h. For this speed, calculate the plane's resultant displacement if it travels east for 1.50 h, then turns 135° north-west and travels for 2.00 h.
5. In 1952, the ocean liner *United States* crossed the Atlantic Ocean in less than four days, setting the world record for commercial ocean-going vessels. The average speed for the trip was 57.2 km/h. Suppose the ship moves in a straight line eastward at this speed for 2.50 h. Then, due to a strong local current, the ship's course begins to deviate northward by 30.0°, and the ship follows the new course at the same speed for another 1.50 h. Find the resultant displacement for the 4.00 h period.