

## Momentum and Collisions

**Problem G****ELASTIC COLLISIONS****PROBLEM**

American juggler Bruce Sarafian juggled 11 identical balls at one time in 1992. Each ball had a mass of 0.20 kg. Suppose two balls have an elastic head-on collision during the act. The first ball moves away from the collision with a velocity of 3.0 m/s to the right, and the second ball moves away with a velocity of 4.0 m/s to the left. If the first ball's velocity before the collision is 4.0 m/s to the left, what is the velocity of the second ball before the collision?

**SOLUTION****1. DEFINE****Given:**

$$m_1 = m_2 = 0.20 \text{ kg}$$

$$\begin{aligned} \mathbf{v}_{1,i} &= \text{initial velocity of ball 1} = 4.0 \text{ m/s to the left} \\ &= -4.0 \text{ m/s to the right} \end{aligned}$$

$$\mathbf{v}_{1,f} = \text{final velocity of ball 1} = 3.0 \text{ m/s to the right}$$

$$\begin{aligned} \mathbf{v}_{2,f} &= \text{final velocity of ball 2} = 4.0 \text{ m/s to the left} \\ &= -4.0 \text{ m/s to the right} \end{aligned}$$

**Unknown:**

$$\mathbf{v}_{2,i} = \text{initial velocity of ball 2} = ?$$

**2. PLAN Choose the equation(s) or situation:** Use the equation for the conservation of momentum to determine the initial velocity of ball 2. Because both balls have identical masses, the mass terms cancel.

$$m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i} = m_1 \mathbf{v}_{1,f} + m_2 \mathbf{v}_{2,f}$$

$$\mathbf{v}_{1,i} + \mathbf{v}_{2,i} = \mathbf{v}_{1,f} + \mathbf{v}_{2,f}$$

**Rearrange the equation(s) to isolate the unknown(s):**

$$\mathbf{v}_{2,i} = \mathbf{v}_{1,f} + \mathbf{v}_{2,f} - \mathbf{v}_{1,i}$$

**3. CALCULATE Substitute the values into the equation(s) and solve:**

$$\mathbf{v}_{2,i} = 3.0 \text{ m/s} - 4.0 \text{ m/s} - (-4.0 \text{ m/s})$$

$$\mathbf{v}_{2,i} = 3.0 \text{ m/s to the right}$$

**4. EVALUATE**

Confirm your answer by making sure that kinetic energy is also conserved.

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

$$v_{1,i}^2 + v_{2,i}^2 = v_{1,f}^2 + v_{2,f}^2$$

$$(-4.0 \text{ m/s})^2 + (3.0 \text{ m/s})^2 = (3.0 \text{ m/s})^2 + (-4.0 \text{ m/s})^2$$

$$16 \text{ m}^2/\text{s}^2 + 9.0 \text{ m}^2/\text{s}^2 = 9.0 \text{ m}^2/\text{s}^2 + 16 \text{ m}^2/\text{s}^2$$

$$25 \text{ m}^2/\text{s}^2 = 25 \text{ m}^2/\text{s}^2$$

**ADDITIONAL PRACTICE**

- The moon's orbital speed around Earth is  $3.680 \times 10^3$  km/h. Suppose the moon suffers a perfectly elastic collision with a comet whose mass is 50.0 percent that of the moon. (A partially inelastic collision would be a much more

realistic event.) After the collision, the moon moves with a speed of  $-4.40 \times 10^2$  km/h, while the comet moves away from the moon at  $-5.740 \times 10^3$  km/h. What is the comet's speed before the collision?

- 2. The largest beet root on record had a mass of 18.40 kg. The largest cabbage on record had a mass of 56.20 kg. Imagine these two vegetables traveling in opposite directions. The cabbage, which travels 5.000 m/s to the left, collides with the beet root. After the collision, the cabbage has a velocity of  $6.600 \times 10^{-2}$  m/s to the right, and the beet root has a velocity of 10.07 m/s to the left. What is the beet root's velocity before the perfectly elastic collision?**
- 3. The first astronaut to walk in outer space without being tethered to a spaceship was Capt. Bruce McCandless. In 1984, he used a jet backpack, which cost about \$15 million to design, to move freely about the exterior of the space shuttle *Challenger*. Imagine two astronauts working in outer space. Suppose they have equal masses and accidentally run into each other. The first astronaut moves 5.0 m/s to the right before the collision and 2.0 m/s to the left afterwards. If the second astronaut moves 5.0 m/s to the right after the perfectly elastic collision, what was the second astronaut's initial velocity? Is the KE conserved?**
- 4. Speeds as high as 273 km/h have been recorded for golf balls. Suppose a golf ball whose mass is 45.0 g is moving to the right at 273 km/h and strikes another ball that is at rest. If after the perfectly elastic collision the golf ball moves 91 km/h to the left and the other ball moves 182 km/h to the right, what is the mass of the second ball?**
5. Jana Novotna of what is now the Czech Republic has the strongest serve among her fellow tennis players. In 1993, she sent the ball flying with a speed of 185 km/h. Suppose a tennis ball moving to the right at this speed hits a moveable target of unknown mass. After the one-dimensional, perfectly elastic collision, the tennis ball bounces to the left with a speed of 80.0 km/h. If the tennis ball's mass is  $5.70 \times 10^{-2}$  kg, what is the target's mass? (Hint: Use the conservation of kinetic energy to solve for the second unknown quantity.)
6. Recall the two colliding snow trains in item 5 of the previous section. Suppose now that the collision between the two trains is perfectly elastic instead of inelastic. The train with a mass of  $4.00 \times 10^5$  kg and a velocity of 32.0 km/h to the right is struck from behind by a second train with a mass of  $1.60 \times 10^5$  kg and a velocity of 36.0 km/h to the right. If the first train's velocity increases to 35.5 km/h to the right, what is the final velocity of the second train after the collision?
7. A dump truck used in Canada has a mass of  $5.50 \times 10^5$  kg when loaded and  $2.30 \times 10^5$  kg when empty. Suppose two such trucks, one loaded and one empty, crash into each other at a monster truck show. The trucks are supplied with special bumpers that make a collision almost perfectly elastic. If the trucks hit each other at equal speeds of 5.00 m/s and the less massive truck recoils to the right with a speed of 9.10 m/s, what is the velocity of the full truck after the collision?