Problem E

TORQUE
PROBLEM

A beam that is hinged near one end can be lowered to stop traffic at a railroad crossing or border checkpoint. Consider a beam with a mass of 12.0 kg that is partially balanced by a 20.0 kg counterweight. The counterweight is located 0.750 m from the beam's fulcrum. A downward force of 1.60×10^2 N applied over the counterweight causes the beam to move upward. If the net torque on the beam is 29.0 N•m when the beam makes an angle of 25.0° with respect to the ground, how long is the beam's longer section? Assume that the portion of the beam between the counterweight and fulcrum has no mass.

SOLUTION

1. **DEFINE**

Given: $m_b = 12.0$ kg

$m_c = 20.0$ kg

$d_c = 0.750$ m

$F_{applied} = 1.60 \times 10^2$ N

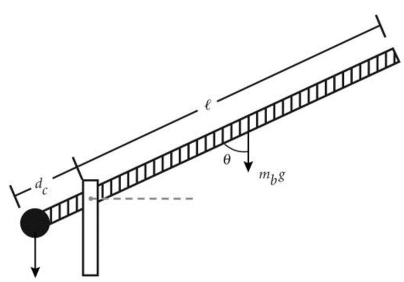
$\tau_{net} = 29.0$ N•m

$\theta = 90.0° - 25.0° = 65.0°$

$g = 9.81$ m/s^2

Unknown: ℓ = ?

Diagram:



2. **PLAN** **Choose the equation(s) or situation:** Apply the definition of torque to each force and add up the individual torques.

$\tau = F\, d\, (\sin\theta)$

$\tau_{net} = \tau_a + \tau_b + \tau_c$

where τ_a = counterclockwise torque produced by applied force = $F_{applied}\, d_c\, (\sin\theta)$

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$$\tau_b = \text{clockwise torque produced by weight of beam}$$
$$= -m_b g\left(\frac{\ell}{2}\right)(\sin\theta)$$
$$\tau_c = \text{counterclockwise torque produced by counterweight}$$
$$= m_c g\, d_c\,(\sin\theta)$$
$$\tau_{net} = F_{applied}\, d_c\,(\sin\theta) - m_b\, g\left(\frac{\ell}{2}\right)(\sin\theta) + m_c\, g\, d_c\,(\sin\theta)$$

Note that the clockwise torque is negative, while the counterclockwise torques are positive.

Rearrange the equation(s) to isolate the unknown(s):

$$m_b g\left(\frac{\ell}{2}\right) = (F_{applied} + m_c g)d_c - \left(\frac{\tau_{net}}{\sin\theta}\right)$$
$$\ell = \frac{2\left[(F_{applied} + m_c g)d_c - \left(\frac{\tau_{net}}{\sin\theta}\right)\right]}{m_b g}$$

3. **CALCULATE Substitute the values into the equation(s) and solve:**

$$\ell = \frac{(2)([1.60\times10^2\ \text{N} + (20.0\ \text{kg})(9.81\ \text{m/s}^2)](0.750\ \text{m}) - \left[\frac{29.0\ \text{N}\bullet\text{m}}{\sin 65.0°}\right]}{(12.0\ \text{kg})(9.81\ \text{m/s}^2)}$$
$$\ell = \frac{(2)[(1.60\times10^2\ \text{N} + 196\ \text{N})(0.750\ \text{m}) - 32.0\ \text{N}\bullet\text{m}]}{(12.0\ \text{kg})(9.81\ \text{m/s}^2)}$$
$$\ell = \frac{(2)[356\ \text{N})(0.750\ \text{m}) - 32.0\ \text{N}\bullet\text{m}]}{(12.0\ \text{kg})(9.81\ \text{m/s}^2)}$$
$$\ell = \frac{(2)(2.67\times10^2\ \text{N}\bullet\text{m} - 32.0\ \text{N}\bullet\text{m}]}{(12.00\ \text{kg})(9.81\ \text{m/s}^2)}$$
$$\ell = \frac{(2)(235\ \text{N}\bullet\text{m})}{(12.0\ \text{kg})(9.81\ \text{m/s}^2)}$$
$$\ell = 3.99\ \text{m}$$

4. **EVALUATE** For a constant applied force, the net torque is greatest when θ is 90.0° and decreases as the beam rises. Therefore, the beam rises fastest initially.

ADDITIONAL PRACTICE

1. **The nests built by the mallee fowl of Australia can have masses as large as 3.00×10^5 kg. Suppose a nest with this mass is being lifted by a crane. The boom of the crane makes an angle of 45.0° with the ground. If the axis of rotation is the lower end of the boom at point *A*, the torque produced by the nest has a magnitude of 3.20×10^7 N•m. Treat the boom's mass as negligible, and calculate the length of the boom.**

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2. **The pterosaur was the most massive flying dinosaur. The average mass for a pterosaur has been estimated from skeletons to have been between 80.0 and 120.0 kg. The wingspan of a pterosaur was greater than 10.0 m. Suppose two pterosaurs with masses of 80.0 kg and 120.0 kg sat on the middle and the far end, respectively, of a light horizontal tree branch. The pterosaurs produced a net counterclockwise torque of 9.4 kN•m about the end of the branch that was attached to the tree. What was the length of the branch?**
3. **A meterstick of negligible mass is fixed horizontally at its 100.0 cm mark. Imagine this meterstick used as a display for some fruits and vegetables with record-breaking masses. A lemon with a mass of 3.9 kg hangs from the**
   70.0 cm mark, and a cucumber with a mass of 9.1 kg hangs from the *x* cm mark. What is the value of *x* if the net torque acting on the meterstick is 56.0 N•m in the counterclockwise direction?
4. In 1943, there was a gorilla named N'gagi at the San Diego Zoo. Suppose N'gagi were to hang from a bar. If N'gagi produced a torque of -1.3×10^4 N•m about point *A*, what was his weight? Assume the bar has negligible mass.
5. The first—and, in terms of the number of passengers it could carry, the largest—Ferris wheel ever constructed had a diameter of 76 m and held 36 cars, each carrying 60 passengers. Suppose the magnitude of the torque, produced by a ferris wheel car and acting about the center of the wheel, is -1.45×10^6 N•m. What is the car's weight?
6. **In 1897, a pair of huge elephant tusks were obtained in Kenya. One tusk had a mass of 102 kg, and the other tusk's mass was 109 kg. Suppose both tusks hang from a light horizontal bar with a length of 3.00 m. The first tusk is placed 0.80 m away from the end of the bar, and the second, more massive tusk is placed 1.80 m away from the end. What is the net torque produced by the tusks if the axis of rotation is at the center of the bar? Neglect the bar's mass.**
7. A catapult, a device used to hurl heavy objects such as large stones, consists of a long wooden beam that is mounted so that one end of it pivots freely in a vertical arc. The other end of the beam consists of a large hollowed bowl in which projectiles are placed. Suppose a catapult provides an angular acceleration of 50.0 rad/s^2 to a 5.00×10^2 kg boulder. This can be achieved if the net torque acting on the catapult beam, which is 5.00 m long, is
   6.25×10^5 N•m.
   a. If the catapult is pulled back so that the beam makes an angle of 10.0° with the horizontal, what is the magnitude of the torque produced by the 5.00×10^2 kg boulder?
   b. If the force that accelerates the beam and boulder acts perpendicularly on the beam 4.00 m from the pivot, how large must that force be to produce a net torque of 6.25×10^5 N•m?

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